The luminosity-redshift relation in brane-worlds: II. Confrontation with experimental data

Gyula M. Szabó, László Á. Gergely, Zoltán Keresztes
Departments of Theoretical and Experimental Physics, University of Szeged, D'om t'er 9, Szeged 6720, Hungary

Abstract. The luminosity distance - redshift relation given analytically for a wide class of generalized Randall-Sundrum type II brane-world models with dark radiation in the companion Paper I is confronted with the presently available supernova data. The procedure selects a class of brane-worlds, fitting slightly better then the ΛCDM model. The model has dark radiation representing up to approximately 5 % of the total energy of the universe and therefore it can be regarded as a slight modification of the ΛCDM model. We show that this model becomes compatible with the known history of the Universe if in previous stages of cosmological evolution the brane radiates away energy into the bulk, thus the dark radiation is only a late-time characteristic. From the supernova data alone the preferred values of the cosmological parameters $\Omega_\rho = 0.225$ and $\Omega_\Lambda = 0.735$ emerge, the former being in full accordance with the WMAP 3-year data. The preferred value of the dark radiation parameter is $\Omega_d = 0.04$.

1. Introduction

Current observational data [1]-[3], suggest that the standard cosmological model based on general relativity and a Universe filled with only baryonic matter has to be modified. The model can be reconciled with observations in the easiest way by the introduction of a cosmological constant Λ and of considerable amount of dark matter (ΛCDM model). As the energy densities of both baryonic and dark matter decrease during cosmological evolution, eventually the cosmological constant will dominate. This process has been recognized after observations of supernovae of type Ia, which suggest that our Universe has reached an accelerating phase. In a Λ-dominated universe, the luminosity distance increases faster with redshift than in the model without Λ [4], exactly as required by the supernova data.

More generically, the agreement with experiments can be achieved by introducing a dark energy component of the Universe, which replaces Λ. Such a dark energy in general does not clump. A recent analysis [5] shows that a dark energy model with varying dark energy density going through a transition from an accelerating to a decelerating phase at redshift 0.45 fits well the observational data. Based on observations, the dark energy equation of state $w = p/\rho$ is within about -1 ± 0.1 [6].

It has been expected for some time that alternative gravitational theories, motivated by string / M-theory could replace dark matter and dark energy by geometric effects. The curved generalizations (see for example the review [7]) of the original Randall-Sundrum type II model [8] consist of a hypersurface with tension λ

(the brane), representing our observable universe, embedded in a 5-dimensional space-time (the bulk). Gravitational dynamics on the brane is governed by an effective Einstein equation [9], [10].

With cosmological symmetries imposed on the brane, the early cosmology is modified [11]. This is due to the source term quadratic in the energy momentum tensor, which at early times, before the Big Bang Nucleosynthesis (BBN) dominates over the linear term. In the simplest case of cosmological symmetries and suppression of the energy exchange between the brane and the bulk and whenever the bulk contains a static black hole, the Weyl curvature of the bulk generates a so-called dark radiation effect on the brane.

Other sources of gravity in the effective Einstein equation include terms due to the asymmetric embedding of the brane into the bulk [10], non-standard model fields in the bulk, and even quantum corrections approximated as induced gravity effects [12]-[15].

Fig 1 of our companion paper [16] (to be referred in what follows as paper I) contains a classification of brane-world theories as well as their inter-relations. They are divided into two branches, one containing the original Randall-Sundrum type II model (BRANE1) and the other the flat DGP model (BRANE2).

As a final test, supernova data were confronted with experiment in the induced gravity models [17]-[20]. When combined with the Sloan Digital Sky Survey (SDSS) data on baryonic oscillations, these seem to rule out [17], [18] the flat DGP models. However it was argued in [20] that the Cosmic Microwave Background (CMB) shift parameter can over-turn this conclusion. Structure formation and CMB were also considered in the DGP models in Ref. [21].

The authors of Ref. [18] have compared the predictions of the flat BRANE1 and BRANE2 models to the Gold [22] and Supernova Legacy Survey (SNLS) [23] supernova data sets, incorporating the baryon acoustic peaks into the analysis. These brane-world models in certain parameter range (when their induced gravity parameter Ω_l is small; the flat DGP models falling outside this range) satisfy both sets of supernova data sets. The BRANE1 models fit better to the SNLS data, while the BRANE2 models fit better to the Gold data set. Since the analysis depends very weakly on the bulk cosmological constant $\widetilde{\Lambda}$, the value of $\widetilde{\Lambda}$ was fixed at zero. Then the BRANE1 model fits better to the SNLS data than the ΛCDM model and fits comparably well to the Gold data. The same conclusion holds for the BRANE2 model. In the analysis of [18] the dark radiation dimensionless parameter Ω_d is switched off.

Using two recent supernova data sets, the CMB shift parameter, and the baryon oscillation peaks, the authors of Ref. [24] have found that the LDGP model (a subclass of the BRANE1 models with the effective energy density having a phantom-like behavior due to extra-dimensional effects, see Fig 1 of Paper I) fits the observations if it is very close to the ΛCDM model. The modification of the LDGP model with respect to the ΛCDM model appears in the form of a linear term in the Friedmann equation, H/r_c, where H is the Hubble parameter and r_c a crossover scale. This model includes a cosmological constant, possibly screened by the modified gravity, however the comparison with observations sets strong constraints on the screening.

The first comprehensive study of the generalized Randall-Sundrum type II (RS) brane-worlds tested against astronomical data was presented in Ref. [25]. Agreement with earlier supernova data has been established in the presence of a cosmological constant. In this analysis the dark radiation from the bulk was switched off ($\Omega_d = 0$) and the energy-momentum squared term was kept. Under these assumptions, for flat spatial sections and matter parameter $\Omega_\rho = 0.3$ the maximum likelihood method gave $\Omega_\lambda = 0.004 \pm 0.016$ for the parameter characterizing the source term quadratic in the energy-momentum. This in turn implies a tiny value of the brane tension, which is disfavored by generic brane-world arguments. Moreover, much lower values for Ω_λ emerge from both CMB and BBN.

In contrast to Refs. [18] and [25] the analysis of [26] keeps both Ω_λ and Ω_d, the latter obeying $|\Omega_d| < 0.01$. The best fit is obtained at $\Omega_\rho = 0.15$, $\Omega_\Lambda = 0.80$, $\Omega_\lambda = 0.026$ and $\Omega_d = 0.008$.

With the high value of the brane tension set by either (a) the value of the 4-dimensional Planck constant and sub-millimeter tests [27] on possible deviations from Newton's law (in units $c = 1 = \hbar$ these give $\lambda^{\min}_{tabletop} = 138.59$ TeV^4 see [28], [7]), (b) astrophysical considerations $\lambda^{\min}_{astro} = 5 \times 10^8$ MeV^4 [29] or (c) BBN constraints $\lambda^{\min}_{BBN} = 1$ MeV^4 [30], the quadratic source term barely counts at late-times in the cosmological evolution.

Given the high limits for the values of λ, in any realistic model Ω_λ can be safely ignored. This is a crucial difference of our forthcoming analysis as compared to the one presented in Refs. [26] and [25], where the corresponding cosmological parameter Ω_λ was kept.

The question then arises whether the source term arising from the Weyl curvature of the bulk may be kept, in other worlds, whether $\Omega_d \neq 0$. The Weyl curvature of the bulk gives an energy density $\rho_d = 6m/\kappa^2 a^4$, where $\kappa^2 = 8\pi G$ is the gravitational coupling constant. In Ref. [31] it was shown that the BBN limits constrained the dark radiation component as $-1.23 \leq \rho_d\left(z_{BBN}\right)/\rho_\gamma\left(z_{BBN}\right) \leq 0.11$. Combining this with CMB constraints reduces this range to $-0.41 \leq \rho_d\left(z_{BBN}\right)/\rho_\gamma\left(z_{BBN}\right) \leq 0.105$. Here ρ_γ is the energy density of the background photons. Another constraint for the value of the dark radiation at BBN was derived in [32] as $-1 < \rho_d\left(z_{BBN}\right)/\rho_\nu\left(z_{BBN}\right) < 0.5$, where ρ_ν is the energy density contributed by a single, two-component massless neutrino. This constraint was derived for high values of the 5-dimensional Plank mass.

With the Weyl source term evolving as radiation, its present value is clearly tiny. This is the reason why all mentioned references [18] and [25] comparing RS brane-worlds with observations disregard dark radiation. But is this truely a necessary assumption?

Formulating the question the other way around: if we include even a small component of dark radiation into the late-time universe model we face a serious problem. Due to the fact that the energy density of dark radiation decreases as a^{-4} (compared to that of matter which is a^{-3}), even an amount of dark radiation of the same order as the amount of baryonic matter nowadays implies dark radiation dominance in the past, for example during structure formation, a conclusion contradicted by numerical simulations, which favorize cold dark matter as the dominant component of the Universe during structure formation [33].

However we can restrict the validity of the model with a *constant* mass m in the dark radiation energy density (see also the definition of Ω_d in Eq. (23.c) of paper I). A constant m implies a static Schwarzschild-anti de Sitter bulk and no energy exchange between the brane and the bulk. Therefore dark radiation is a manifestation of an equilibrium configuration with a static bulk, and it may be well possible that such a situation is reached only at the latest stages of the evolution of the brane-world Universe. Whenever m depends on certain non-zero power of a, the evolution of the energy density of the Weyl source term evolves in a non-standard way, allowing to escape from the argument of a small dark radiation left nowadays.

We propose here the LLDRRS (Lambda-Late-time-Dark-Radiation-Randall-Sundrum) model, a specific *RS model with i) cosmological constant, ii) the brane radiating away energy in the earlier stages of cosmological evolution, and iii) a late-time dark radiation characteristic for the latest stage of cosmological evolution.* The latter stage can be tested by supernova observations. For the inclusion of the LLDRRS model in the classification of brane-world models, see Fig 1 of paper I.

The LLDRRS model takes into account the possibility of an energy exchange between the brane and the bulk. This idea is not new. Indeed, it was already proposed that during an inflationary phase on the brane radiation is emitted and black holes thermally nucleate in the bulk [34]. Later on, but still in the high energy regime, the brane radiates such that the *mass function* of the bulk black hole increases with a^4 [35]. This means that the Weyl source term becomes a constant in this era. The brane continues to radiate away energy during structure formation [36], a process leading to a bulk black hole mass function $m \propto a^{\alpha}$, with $1 \leq \alpha \leq 4$. (Other models with the brane radiating energy into the bulk are also known [37].) Therefore pure dark radiation emerges only in the low-z limit, while at earlier times a dynamic bulk - brane interaction governed by energy exchange could be present.

In this paper we compare the predictions of both the LLDRRS model and various other brane-world models of the RS class with the available supernova data. In the process we employ the analytical results of paper I.

Checking if a cosmology is compatible with the supernova data implies to fit the predicted $d_L(z)$ curve (luminosity distance d_L as function of redshift z) to the observed d_L–z data pairs. This process includes the determination of the cosmological parameters.

As type Ia supernovae result from the explosion of white dwarf stars with identical mass, they show remarkable similarities. By employing well established calibration methods, one can calculate the maximal luminosity of the object (in the reference system of the explosion). This is done by analyzing the time-dependent variation of the emitted luminosity and the spectrum, a method known as the Multi-Color Light Curve analysis [38], [22]. In this process the observed parameters, the *shape of the light curve* and the *spectral distribution of the emission* have to be converted into the reference system of the host galaxy. For distant supernovae this translates to take into account the time dilation and the so-called K-correction [39]. While these methods depend on z, they are independent on the specific cosmological model. After performing these corrections, we have well-calibrated maximal luminosities for the supernovae of type Ia and in consequence they are considered as standard candles.

In 2003 a list of d_L–z data pairs were published for 230 supernovae of type Ia [40]. We first select a *comparable* subset from this data and compare them with the predictions of the enlisted models in section 2. On this ground (in accordance with general expectations based on the running of the dark radiation energy density with the negative fourth power of the scale factor) we can eliminate the model (A) and one of the models (B), but surprisingly a good fit is obtained for the other brane-world model (B). This model however cannot represent our Universe, as its very low brane tension is disfavored by generic brane-world arguments. A remarkable fit is obtained for the models (C), with both small negative and positive values of the dark radiation.

In 2004 the Gold set of data [22] was released. There is also another data set available, from the first year run of the Supernova Legacy Survey Measurement [23], which consists of 71 medium to high redshift supernovae, complemented by recalibrated earlier data. We chose the Gold set for further analysis for two reasons. First, we have found sensible differences in how the error bars of the SNLS and Gold data sets compare to their scatter. More specifically, the χ^2-test lead to values considerably exceeding the critical ones, likely due to the small error bars of the low-z data from the SNLS set. Second, the supernovae with highest z is approximately 1.5 times farther in the Gold set, and the cosmological model is obviously influenced by these distant supernovae.

In section 3 we compare the remaining models with the Gold data set, first by employing the data set as is (subsection 3.1), then based on the smeared set of Gold data (subsection 3.2). The latter analysis eliminates the models (C) with significant negative values of the dark radiation, leaving the LLDRRS model with a small amount of positive dark radiation as the most viable candidate. These models show an excellent fit, some of them better than the fit in the ΛCDM model. For example the χ^2-test for the LLDRRS model with $\Omega_d = 0.05$ shows a 4 % improvement over the ΛCDM model.

An improved Gold data set [41] was released in 2006. In section 4, based on this more recent set we repeat the analysis of the previous section, ruling out the second model (B) on the grounds of supernova observations as well. Then we perform an optimization of the still surviving LLDRRS models in the parameter space $\Omega_d - \Omega_\rho$ (and $\Omega_\Lambda - \Omega_\rho$) and we find the values preferred by supernova data for these cosmological parameters.

We proceed in section 5 by reconciliating the LLDRRS model with the known history of the Universe. We explicitly show here that the preferred value of dark radiation becomes possible if the brane radiates even for a relatively short period during the cosmological evolution.

Finally section 6 contains the concluding remarks.

Although the analytical results of paper I were given for $c = 1$, the SI value of c was reintroduced for all plots and numerical estimates in this paper.

2. Confronting the models with the selected low absorption supernova data

We confront with supernova observations several models from paper I. In Fig 1 we represent graphically on both logarithmic and linear scales their luminosity distance - redshift relations up to $z = 2.5$. The plots are for $k = 0$ and $\Omega_\rho = 0.27$ (according to the combined analysis of the SDSS and WMAP 1-year data in Ref [2]). In particular, the luminosity distance - redshift relation is shown for the following models:

- The LLDRRS model (the perturbative solution given by Eqs. (53), (57)-(58), (60)-(61) of paper I, with $\Omega_\lambda = 0$) for the two values of the late-time dark radiation $\Omega_d = -0.05$ and $\Omega_d = 0.05$ (the curves 1 and 3, respectively). The latter models contain a brane which radiates away energy at early times (for $\Omega_d > 0$) and during structure formation, such that a bulk black hole is formed and its mass increases continuously. As this process slows down, the Weyl curvature of the bulk induces the late-time dark radiation on the brane.
- The ΛCDM model, given by Eqs. (57)-(58) of paper I (curve 2).
- The solutions with brane tension $\lambda = 2\Lambda/\kappa^2$ and no dark radiation (given by Eq. (49) of paper I) for both admissible values for this model, at $\Omega_\Lambda = 0.704$ (curve 4)

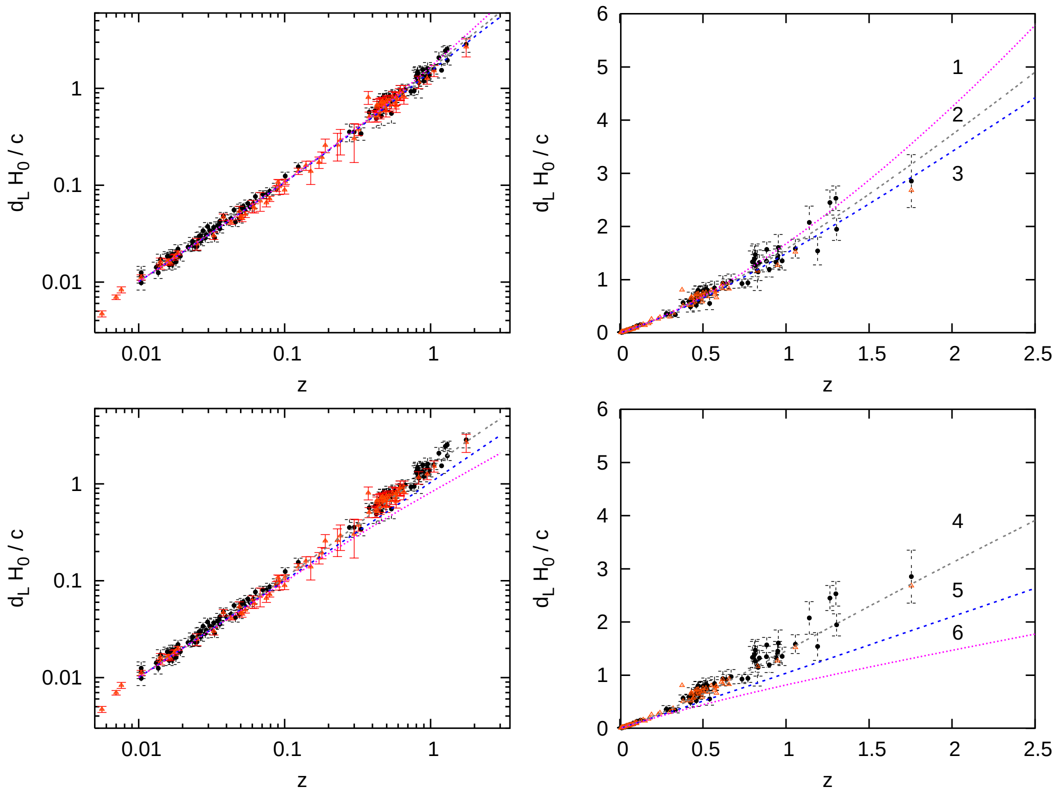

Figure 1. (Color online) Luminosity distance – redshift relations for selected brane-world cosmologies and for the ΛCDM model, compared to the supernova data. The diagrams are log-scaled (left panel) and linearly scaled (right panel). Selected low absorption supernova measurements (as discussed in the text) from Ref. [40] are plotted with red, black dots represent the Gold set [22]. Both sets are represented with the corresponding error bars on the log-scaled diagrams. For the sake of perspicuity, the error bars of low absorption supernovae are not represented on the linearly scaled diagrams. The plotted models are the ΛCDM model (2); the brane models with cosmological constant and late-time dark radiation (1 and 3); without cosmological constant but with dark radiation (5); with cosmological constant satisfying $\Lambda = \kappa^2\lambda/2$, thus low brane tension (4 and 6) and no dark radiation. More specifically, the plots refer to luminosity distances with (1) $\Omega_\lambda = 0$ and $\Omega_d = -0.05$; (2) $\Omega_\lambda = 0 = \Omega_d$; (3) $\Omega_\lambda = 0$ and $\Omega_d = 0.05$; (4) $\Omega_\Lambda = 0.704$ and $\Omega_\lambda = 0.026$; (5) $\Omega_\Lambda = 0$ and $\Omega_d = 0.73$; (6) $\Omega_\Lambda = 0.026$ and $\Omega_\lambda = 0.704$. In all plots $\Omega_\rho = 0.27$ was assumed and $\Omega_\rho + \Omega_\Lambda + \Omega_d + \Omega_\lambda = 1$ holds.

and $\Omega_\Lambda = 0.026$ (curve 6). The former is similar to the class of models discussed in [25].

- The late-time universe $\Omega_\lambda = 0$ limit of the RS model with Randall-Sundrum fine-tuning, containing a huge amount of dark radiation $\Omega_d = 0.73$, given by Eq. (41) of paper I (curve 5).

Then we compare the model-dependent luminosity distances with the distant supernova data, as follows. On the same graphs we plot a set of selected supernova data from Ref. [40] (red triangles) together with the Gold set [22] (black dots). The error bars are indicated in the respective colors. The diagrams with linear scale are more instructive, as they emphasize the difference among the predictions of the chosen models and how they fit data, while the logarithmic scale better disseminate between the low z points.

The models represented by the curves 1, 3 and 4 by eye seem to compare as well with the supernova observations as the ΛCDM model (curve 2). By contrast, the models represented by the curves 5 and 6 seem to be not supported by observations.

In order to further discern among the models, in this subsection we apply a χ^2-test to the low absorption sample of supernovae (a set of 79 data points) given in Ref. [40]. The test consists of calculating the sum of squares of the deviances between the n measures L_i and the model prediction for the data points M_i, divided by the square of the error bar $\sigma\left(L_i\right)$,

$$\chi^2 = \sum_{i=1}^{n} \frac{\left(L_i - M_i\right)^2}{\sigma^2\left(L_i\right)} \ . \tag{1}$$

If the data set is *comparable* and the model fits well, the sum is smaller than the *critical value* of the χ^2-statistics of a given significance level and of $(n-p)$ degrees of freedom (where p is the number of parameters to be determined from the model, $p \ll n$).

First we have found that for the ΛCDM model the low-absorption sample *fails* to be comparable, as χ^2 is much too large. Therefore we have dropped out all 18 low-z points ($z < 0.01$), which show considerable scatter. This procedure is also motivated by the fact that the predictions of the models discussed here are different only for higher z-s. We have also dropped out a point (SN 1997O) near $z = 0.4$ for which $d_L H_0/c$ had a much higher value of 0.87 as compared to $d_L H_0/c \approx 0.5 \pm 0.1$ for other data at $z \approx 0.4$.

The remaining Selected Low Absorption (SLA) sample contains 60 points. The critical χ^2 values for $n = 60$ and a significance level of 5 %, 1 % and 0.1 % are 79, 88 and 100, respectively. The canonical model ($\Omega_\rho = 0.27$ and $\Omega_\Lambda = 0.73$) gives $\chi^2 = 51$, showing that the SLA sample is comparable, and its fit with the ΛCDM model (curve 2 of Fig 1) is good, as expected.

The model with no cosmological constant and significant dark radiation $\Omega_d = 0.73$, $\Omega_\lambda = 0$ (curve 6) and the model with $\Lambda = \kappa^2\lambda/2$ and $\Omega_\Lambda = 0.025$ (curve 5) are significantly inconsistent with the observations, as they give $\chi^2 = 213$ and 395, respectively‡. All other models shown on Fig 1 are comparable with the supernova observations, as it was expected by a simple glance.

The $\chi^2 = 50$ value found for the $\Lambda = \kappa^2\lambda/2$-model with $\Omega_\Lambda = 0.74$ (curve 4) is slightly better than χ^2 found the ΛCDM model. However, as mentioned earlier, the tiny brane tension $\lambda = 38.375 \times 10^{-60}\mathrm{TeV}^4$, several order of magnitudes lower than all existing lower limits rules out this model as well.

The best fitting models are the models with brane cosmological constant; a high value of the brane tension (leading to $\Omega_\lambda \approx 0$) and a small contribution of dark radiation, $\Omega_d = \pm 0.05$ (the curves 1 and 3). For $\Omega_d = -0.05$ we find $\chi^2 = 65$, which is still acceptable. For $\Omega_d = 0.05$ we get $\chi^2 = 49$ that is almost 4 % better than for the ΛCDM solution.

‡ We mention here that we have also excluded several other models with Randall-Sundrum fine-tuning (not shown on Fig 1), which have either a very low value of the brane tension or a significant dark radiation. For example, the models with $\Omega_d = 0.0258835$, $\Omega_\lambda = 0.70412$ and $\Omega_d = 0.70412$, $\Omega_\lambda = 0.0258835$ gave $\chi^2 = 246$ and 415, respectively.

Values of Ω_d between these limits are also admissible. It is likely that by increasing Ω_d towards higher positive values, χ^2 remains compatible, however the accuracy of the perturbative solution is deteriorated with increasing Ω_d, therefore higher orders in the expansion would be necessary to take into account.

3. Gold2004 set of supernovae

3.1. Analysis base on the data

The Gold set [22] consist of 157 points, including about half of the supernovae listed in Ref. [40] and additional data, merely measurements of the Hubble Space Telescope (HST).

Using the Gold set we have again tested the brane models compatible with the SLA data (the curves 1-4 of Fig 1). The 5 %, 1 % and 0.1 % level critical values for $n = 157$ are 195, 213 and 235. The calculation proceeded in a similar fashion as in the previous section.

The result though are slightly different. The lowest χ^2 was found this time for the ΛCDM model (curve 2), with the value 158, and the highest ($\chi^2 = 178$) for the low brane-tension model represented by the curve (4). The dark radiation models (curves 1 and 3) got almost identical intermediate-valued χ^2-s, e.g. 167 and 168 (for $\Omega_d = -0.05$ and $+0.05$), respectively.

We conclude that the Gold set prefers the steeper (more accelerating) cosmological models as compared to the SLA data. However, we should emphasize that the differences of these numerical values do not enable us to firmly state that either of the models represented by the curves 1-4 is superior to the others, statistically they are similarly compatible.

3.2. Smeared data from the Gold set of supernovae

The possible reason for the difference in the outcome of the previous two data analyses is that the Gold set has many data points between $z = 0.1$ and $z = 0.3$, which lay slightly above the model curves. This can result in a preference for steeper luminosity-redshift functions. The high-z points of the Gold set lay slightly under the ΛCDM solution but they have larger error bars and lighter weights σ^{-2}. In order to balance the medium and high redshift parts of the Gold set, we have smeared the data into 0.1 z-bins, exactly as proposed in Ref. [22]. The weighted average of redshift $\langle z\rangle$, the weighted average of luminosity distance $\langle d_L\rangle_{\langle z\rangle}$ and the resulting error bars $\langle\sigma(d_L)\rangle_{\langle z\rangle}$ are then given as

$$\langle z\rangle = \left[\sum_{i\,\in\, z-\mathrm{bin}} \frac{z_i}{\sigma^2(d_{L,i})}\right]\left[\sum_{i\,\in\, z-\mathrm{bin}} \frac{1}{\sigma^2(d_{L,i})}\right]^{-1}, \tag{2}$$

$$\langle d_L\rangle_{\langle z\rangle} = \left[\sum_{i\,\in\, z-\mathrm{bin}} \frac{d_{L,i}}{\sigma^2(d_{L,i})}\right]\left[\sum_{i\,\in\, z-\mathrm{bin}} \frac{1}{\sigma^2(d_{L,i})}\right]^{-1}, \tag{3}$$

$$\langle\sigma(d_L)\rangle_{\langle z\rangle} = \left[\sum_{i\,\in\, z-\mathrm{bin}} \frac{1}{\sigma^2(d_{L,i})}\right]^{-1/2}, \tag{4}$$

where the summation index i runs over all data from a given z-bin $\langle z\rangle$.

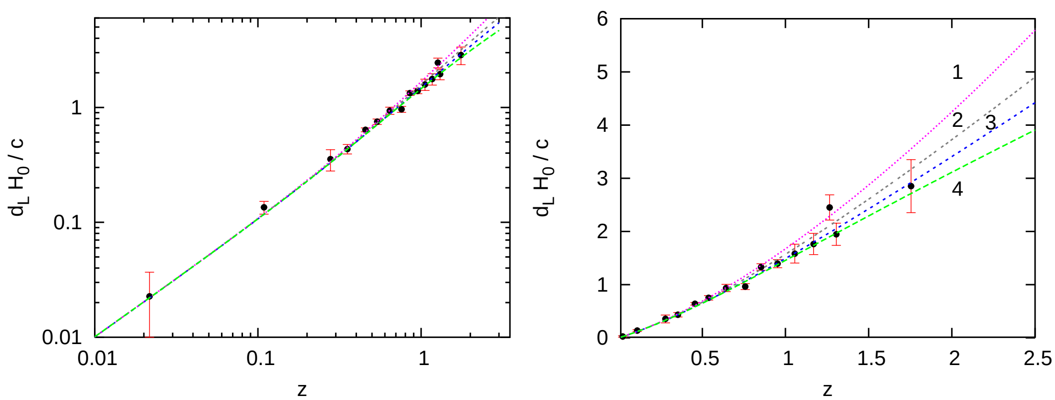

Figure 2. (Color online) The luminosity distance - redshift relation for the viable brane-world models and the ΛCDM model (the curves (1)-(4) of Fig 1), both with logarithmic (left panel) and linear scale (right panel), compared to the smeared Gold set [22]. The best fit is obtained for the brane-world model (3), with 5% dark radiation.

The smeared data set has 15 points (corresponding to 15 z-bins), plotted in Fig 2. The 5 %, 1 % and 0.1 % level critical values of χ^2 are 25, 30 and 37 for the 15 points.

Remarkably, the smeared set disfavors the dark radiation model with $\Omega_d = -0.05$ (curve 1) as $\chi^2 = 37.0$. The low brane tension model (curve 4) gives 25.4. The ΛCDM and the positive dark radiation models (curves 2 and 3) have the lowest χ^2 values of 20.0 and 19.2, respectively. This, once more means that the RS model with 5 % dark radiation is 4 % better than the ΛCDM.

Again, the brane-world model with dark radiation has the best fit, as in the case of the SLA data. However, due to the low number of data one should consider the models represented by the curves 2-4 as statistically equally possible.

4. Gold2006 set of supernovae

More recently, Riess et al. [41] have published a new set of 182 gold supernovae, including new HST observations and recalibrations of the previous measurements. It is an interesting question how this recalibration influenced the above conclusions for the well-fitting models with dark radiation.

We applied the same tests to the Gold2006 data set as described in the previous section. First we assumed that $\Omega_\rho = 0.27$, as before, cf. Ref [2]. In this case the critical value of χ^2 is 197. Then the models represented by the curves 1-4 of Fig 2 behave as follows. The model (C) with negative dark radiation is disfavored once more ($\chi^2 = 204$). So is this time the model (B) with $\lambda = 2\Lambda/\kappa^2$ and $\Omega_\Lambda = 0.704$ (as $\chi^2 = 221$). As expected from the previous analysis, the ΛCDM model ($\chi^2 = 192$) and the LLDRRS model with $\Omega_d = 0.05$ (giving $\chi^2 = 194$) compete closely.

We also remark that varying Ω_d between -0.03 and 0.07, the χ^2 remains under the critical value.

For gaining a deeper insight we have then calculated the predictions of the models between $\Omega_d = -0.10 \div 0.10$ with a stepsize of 0.01 in Ω_d, with Ω_ρ allowed to freely vary in the domain $0.15 \div 0.35$ and z in the range $0 \div 3$. Then we looked for the

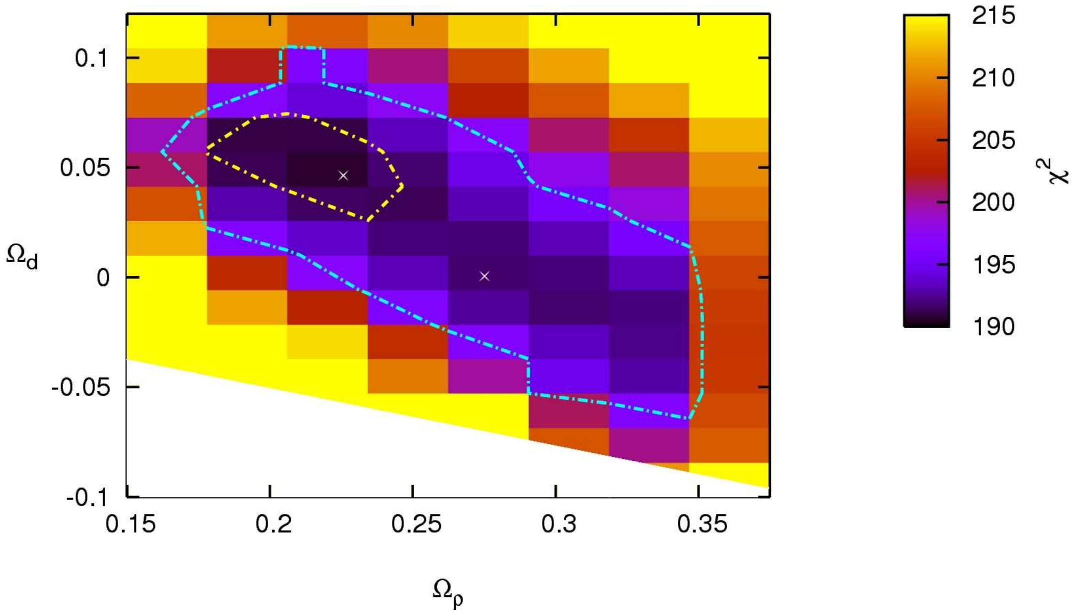

Figure 3. (Color online) The fit of the luminosity distance - redshift relation for the LLDRRS brane-world models with dark radiation, (including the ΛCDM model for $\Omega_d = 0$). There is no assumption for $\Omega_\rho \in (0.15,\ 0.35)$, its preferred value 0.225 being determined from the supernova data, together with the preferred value 0.040 of Ω_d. The contours refer to the 1-σ and 2-σ confidence levels and both are centered on the LLDRRS model with the values given above. The local minimum represented by the ΛCDM model is at $\Omega_\rho = 0.275$. Both the global and local minima are marked. The white area in the lower left corner represents the forbidden region of the parameter space for $z = 3$.

best fit of the Gold2006 set in the $\Omega_d - \Omega_\rho$ space. This is represented on Fig 3. The global minimum of the surface is at $\Omega_d = 0.040$, $\Omega_\rho = 0.225$ ($\chi^2 = 190.52$), which suggests an interesting opportunity for a Universe with less baryonic density and with dark radiation, compatible with the Gold2006 supernova data. The 1-σ confidence interval is centered about this value. The ΛCDM model (where Ω_d is exactly 0) has the local minimum of $\Omega_\rho = 0.275$ ($\chi^2 = 195.8$), but this is outside the 1-σ confidence interval. The global minimum of the LLDRRS model is almost 3 % better than the local minimum of the ΛCDM model.

Similar conclusions emerge from the plot in the $\Omega_\Lambda - \Omega_\rho$ plane, Fig 4. Here the global minimum of the surface is at $\Omega_\Lambda = 0.735$, $\Omega_\rho = 0.225$.The local minimum of the ΛCDM model is at $\Omega_\Lambda = 0.725$.

We note that there is a forbidden parameter range in both planes $\Omega_d - \Omega_\rho$ and $\Omega_\Lambda - \Omega_\rho$, represented by white regions on Figs 3 and 4. This is because the Friedmann equation for these brane-world models

$$\left[\frac{H\left(z\right)}{H_0}\right]^2 = \Omega_\Lambda + \Omega_\rho \left(1+z\right)^3 + \Omega_d \left(1+z\right)^4 > 0, \tag{5}$$

combined with $\Omega_\Lambda + \Omega_\rho + \Omega_d = 1$ gives the constraints

$$\Omega_d \left[\left(1+z\right)^4 - 1\right] + \Omega_\rho \left[\left(1+z\right)^3 - 1\right] + 1 > 0 \tag{6}$$

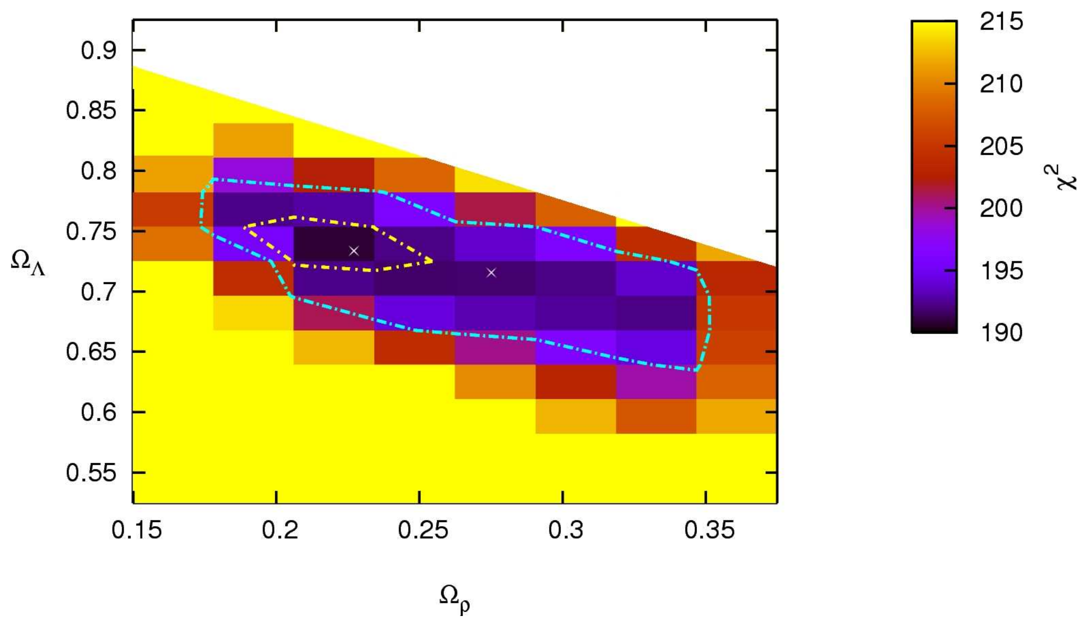

Figure 4. (Color online) Same as on Fig 3, but in the $\Omega_\Lambda - \Omega_\rho$ plane. The global minimum is at $\Omega_\Lambda = 0.735$, $\Omega_\rho = 0.225$, while the local minimum for the ΛCDM model gives $\Omega_\Lambda = 0.725$ and $\Omega_m = 0.275$ (both marked). The white area on the top right corner represents the forbidden parameter range.

in the $\Omega_d - \Omega_\rho$ plane and

$$\Omega_\Lambda \left[(1+z)^4 - 1 \right] - (1+z)^3 \left[1 + z \left(1 - \Omega_\rho \right) \right] < 0 \tag{7}$$

in the $\Omega_\Lambda - \Omega_\rho$ plane.

The forbidden region increases in both cases with z. If we would like to extend the limits to $z \to \infty$, we obtain the limiting curves $\lim_{z\to\infty} \Omega_d^{\min} (z, \Omega_\rho) = 0$ in the $\Omega_d - \Omega_\rho$ plane and $\lim_{z\to\infty} \Omega_\Lambda^{\max} (z, \Omega_\rho) = 1 - \Omega_\rho$ in the $\Omega_\Lambda - \Omega_\rho$ plane. However the LLDRRS model being valid only for low values of z, we represent on the graphs only the forbidden range for $z = 3$.

5. The compatibility of $\Omega_d = 0.04$ with cosmological evolution in the LLDRRS model

The constraint derived in [31] for the energy density of the dark radiation:

$$-0.41 \leq \frac{\rho_d \left(z_{BBN} \right)}{\rho_\gamma \left(z_{BBN} \right)} \leq 0.105, \tag{8}$$

where $\rho_\gamma \left(z_{BBN} \right) = \beta T_{BBN}^4$ is the energy density of the background photons at the beginning of BBN. The coefficient

$$\beta = \frac{\pi^2}{30} g_* \frac{k_B^4}{(\hbar c)^3} = 3.78 \times 10^{-16} \ g_* \text{ J m}^{-3} \text{ K}^{-4} \tag{9}$$

contains [11], [42] the effective number g_* of relativistic degrees of freedom, which depends on the temperature. According to [43] $g_* = 10.75$ at the beginning of BBN, when $T_{BBN} = 1.16 \times 10^{10}$ K. Thus $\rho_\gamma(z_{BBN}) = 7.37 \times 10^{25}$ J m^{-3} emerges, giving the constraint

$$-3.02 \times 10^{25}\ \mathrm{Jm}^{-3} \leq \rho_{\mathrm{d}}(\mathrm{z}_{\mathrm{BBN}}) \leq 7.74 \times 10^{24}\ \mathrm{Jm}^{-3}. \tag{10}$$

Note, that the domain of allowable negative values is larger than the one for positive values.

As for today the background photons have cooled to $T_0 = 2.725$ K and for such low temperatures $g_* = 3.36$ [42], [43] their energy density $\rho_\gamma = \rho_\gamma(z=0)$ is

$$\rho_\gamma = 7.01 \times 10^{-14}\mathrm{Jm}^{-3}. \tag{11}$$

With the value $H_0 = 73^{+3}_{-3}$ km s^{-1} Mpc^{-1} of the Hubble constant [3], cf. Eq. (23) of paper I the present day cosmological parameters ρ and Ω (both for background and dark radiation) relate as

$$\rho_{d,\gamma} = 9.00 \times 10^{-10}\ \Omega_{d,\gamma}\mathrm{Jm}^{-3}. \tag{12}$$

Thus the present value of Ω_γ is

$$\Omega_\gamma = 7.74 \times 10^{-5}, \tag{13}$$

which is quite negligible. If the Weyl source term were to evolve as radiation, its value would be even smaller, cf. Eq. (8). Indeed Eqs. (10) and (12) imply

$$-1.02 \times 10^{-4} \leq \Omega_d \leq 2.62 \times 10^{-5}. \tag{14}$$

$|\Omega_d|$ is of the same order of magnitude or smaller as Ω_γ.

However if the brane is radiating during structure formation, the mass parameter m becomes a function of the scale factor $m \propto a^\alpha$, with $1 \leq \alpha \leq 4$ [36]. Then the energy density scales as $a^{4-\alpha}$.

Now let us suppose that the brane is in an equilibrium (non-radiating) configuration with $\alpha = 0$ in the domain $0 \leq z \leq z_1$. In a preceding era $z_1 < z \leq z_*$ the brane radiates such that $\alpha \neq 0$, finally right after the beginning of BBN, at $z_* < z \leq z_{BBN}$ there is equilibrium once more ($\alpha = 0$). Here $z_{BBN} = (T_{BBN}/T_0) - 1 = 4.26 \times 10^9$. According to this evolution

$$\begin{aligned}\rho_d(z_{BBN}) &= \rho_d \left(\frac{a_0}{a_1}\right)^4 \left(\frac{a_1}{a_*}\right)^{4-\alpha} \left(\frac{a_*}{a_{BBN}}\right)^4 \\ &= \rho_d \left(\frac{1+z_1}{1+z_*}\right)^\alpha (1+z_{BBN})^4 .\end{aligned} \tag{15}$$

Inserting this in Eq. (10) and employing Eq. (12) we obtain:

$$-1.02 \times 10^{-4} \leq \left(\frac{1+z_1}{1+z_*}\right)^\alpha \Omega_d \leq 2.62 \times 10^{-5} . \tag{16}$$

In the particular case $\alpha = 0$ we recover the constraint (14) set on pure dark radiation. However for any $\alpha > 0$ we get

$$z_* \geq (1+z_1)\ \ [\max(-0.98\ \Omega_d,\ 3.82\ \Omega_d)]^{1/\alpha} \times 10^{4/\alpha} - 1. \tag{17}$$

Let us specify this result for the best fit value $\Omega_d = 0.04$. Depending on α we obtain the following numerical relations between the redshifts characterizing the swithcing on and off of the radiation leaving the brane:

$$z_* \geq \begin{cases} 1527.80 + 1528.80\ z_1 & , \quad \alpha = 1 \\ 38.10 + 39.10\ z_1 & , \quad \alpha = 2 \\ 10.52 + 11.52\ z_1 & , \quad \alpha = 3 \\ 5.25 + 6.25\ z_1 & , \quad \alpha = 4 \end{cases} . \tag{18}$$

It is evident that the value of z_* increases with z_1 (this dependence becoming an approximate scaling for higher values of z_1) and decreases with α.

The lower limit in the LLDRRS model is $z_1 = 3$. Then

$$z_* \geq \begin{cases} 6114.20 & , \quad \alpha = 1 \\ 155.40 & , \quad \alpha = 2 \\ 45.08 & , \quad \alpha = 3 \\ 24.01 & , \quad \alpha = 4 \end{cases} . \tag{19}$$

For the higher values of α the duration of the radiative brane regime necessary to produce a high value of Ω_d today is quite short.

6. Concluding remarks

The luminosity distance given in paper I as function of redshift in terms of elementary functions and elliptical integrals of first and second type for various brane-world models with dark radiation was confronted with the available supernova data sets. The tested models were:

(A) The models with Randall-Sundrum fine-tuning, discussed in section 4 of paper I, with a considerable amount of dark radiation as a bulk effect, and a high value of the brane tension.

(B) The two models discussed in subsection 5.1 of paper I, which obey $\Lambda = \kappa^2\lambda/2$, have no dark radiation and were integrable in terms of elementary functions.

(C) The LLDRRS models (subsection 5.2 of paper I), with a brane cosmological constant, for which the luminosity distance could be given analytically as function of redshift to first order accuracy in the dark radiation. (Due to its smallness, the source term Ω_λ quadratic in the energy density was suppressed in the perturbative models of paper I.)

We have carried out the comparison first with the Selected Low Absorption (SLA) sample selected from the data of Ref. [40], second with the Gold data [22] and third with the improved Gold2006 data [41].

In the brane-world model (A) we have introduced an extremely high amount of dark radiation $\Omega_d = 0.73$, tentatively replacing the cosmological constant in the energy balance $\Omega_\Lambda + \Omega_\rho + \Omega_d + \Omega_\lambda = 1$. This model was quickly outruled by supernova data (curve 5 of Fig 1). Dark radiation is not capable to replace the cosmological constant in producing a late-time acceleration. This conclusion is not surprising, since dark radiation scales as usual radiation. The more we go back in the past, the higher becomes its domination over matter. Therefore a cosmological constant or dark energy is still needed in the generalized Randall-Sundrum type II models.

Our analysis has also dismissed immediately the model (B) with $\Omega_\Lambda = 0.026$. Surprisingly, the other toy model (B) with $\Omega_\Lambda = 0.704$ was in good agreement with both the SLA and Gold supernova data, but was finally ruled out by the Gold2006 data. This model also suffers from a low value of the brane tension, as the models discussed in Ref. [25], and in consequence it is ruled out by various upper limits set for the brane tension by cosmological and astrophysical tests as well.

The perturbative approach of subsection 5.2 of paper I can be considered valid for a dark radiation $-0.1 < \Omega_d < 0.1$. In this range the LLDRRS brane-world models (C) were confronted with supernova data and the dark radiation with significant negative energy density ruled out.

The remaining LLDRRS brane-world models with Ω_d between -0.03 and 0.07 (and Ω_Λ changed accordingly) turned out to be excellent candidates for describing our universe, as they show remarkable agreement with either the SLA, Gold or Gold2006 supernova data sets.

More specifically, a remarkable agreement (essentially comparable with the fit of the ΛCDM model, however 4 % better for the Gold set) was found for the LLDRRS model with curvature index $k = 0$, containing baryonic and dark matter as $\Omega_\rho = 0.27$, a cosmological constant represented by $\Omega_\Lambda = 0.68$, a high value of the brane tension (leading to $\Omega_\lambda \approx 0$) and a small positive contribution of late-time dark radiation, $\Omega_d = 0.05$. The Gold2006 data had the same preference for the LLDRRS model over the ΛCDM model.

A further statistical analysis based on the Gold2006 data has shown that if Ω_ρ is allowed to vary in the range (0.15, 0.35), the preferred values are $\Omega_d = 0.040$, $\Omega_\rho = 0.225$, $\Omega_\Lambda = 0.735$.

We must note that the reliability of these values is somehow deteriorated by the relatively small number of high-z supernova and by the inherent difficulties in the calibration of the available supernova data. An obvious source of error is that data from the Gold2006 set is a combination of measurements taken on different instruments [44] and in fact it has been already signaled that the Gold2006 data set is not statistically homogeneous [45].

However we stress that the preferred cosmological parameters determined by comparing the LLDRRS model with supernova data alone are in perfect accordance with the WMAP 3-year data. Indeed according to Ref. [3] $\Omega_\rho h^2 = 0.127^{+0.007}_{-0.013}$ and $h = 0.73^{+0.03}_{-0.03}$ from which $\Omega_\rho = 0.238^{+0.035}_{-0.041}$ emerge. The value of Ω_ρ determined by comparing the LLDRRS model with the supernova data alone is well in the middle of the domain allowed by the WMAP 3 year data.

We have then proved that the preferred value of $\Omega_d = 0.04$ is compatible with the known history of the Universe if the brane radiates away energy into the bulk during a relatively short period of the cosmological evolution. Such a process occuring between $z = 24$ and $z = 3$ could increase the amount of dark energy today with a factor of 10^3 as compared to the non-radiating brane, exactly as required by the LLDRRS model.

The fact that a positive dark radiation (corresponding to a bulk black hole rather than to a bulk naked singularity) is favoured by the presently available best supernova data is in accordance with the early behavior of the RS model with late-time dark radiation, where the brane radiating away energy in early times leads to a black hole, which can further grow during structure formation

The difference between the predictions of the two acceptable models of our analysis (the ΛCDM model with $\Omega_d = 0$ and the LLDRRS brane-world with $\Omega_d = 0.04$) are increasing with z (see Figs 1, 2). One may reasonably hope that the very far ($z > 2$) supernovae, which will be discovered for sure in the following decade, will improve their comparison.

Acknowledgments

We thank Botond Nagy for interactions in the early stages of this research. This work was supported by OTKA grants no. T046939, TS044665 and T042509. LÁG was further supported by the János Bolyai Grant of the Hungarian Academy of Sciences and GyMSz by the Magyary Zoltán Higher Educational Public Foundation.

References

[1] Doroshkevich A, Tucker D L, Allam S and Way M J, *Large scale structure in the SDSS galaxy survey* 2004 *Astron. Astrophys.* **418** 7

[2] Tegmark M, Strauss M A, Blanton M R et al., *Cosmological parameters from SDSS and WMAP* 2004 *Phys. Rev.* D **69** 103501

[3] Spergel D N, Bean R, Doré O et al., *Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology* 2006 *astro-ph/0603449*

[4] Sahni V and Starobinski A, *The Case for a Positive Cosmological Lambda-term* 2000 *Int. J. Mod. Phys.* D **9** 373

[5] Fay S and Tavakol R, *A model-independent dark energy reconstraction schem using the geometrical form of the luminosity-distance relation* 2006 *Phys. Rev.* D **74** 083513

[6] Krauss L M, Jones-Smith K, Huterer D, *Dark Energy, A Cosmological Constant, and Type Ia Supernovae* 2007 *astro-ph/0701692*

[7] Maartens R, *Brane-world Gravity* 2004 *Living Rev. Rel.* **7** 1

[8] Randall L and Sundrum R, *An Alternative to Compactification* 1999 *Phys. Rev. Lett.* **83** 4690

[9] Shiromizu T, Maeda K and Sasaki M, *The Einstein Equations on the 3-Brane World* 2000 *Phys. Rev.* D **62** 024012

[10] Gergely L Á, *Generalized Friedmann branes* 2003 *Phys. Rev.* D **68** 124011

[11] Binétruy P, Deffayet C, Ellwanger U and Langlois D, *Brane cosmological evolution in a bulk with cosmological constant* 2000 *Phys. Lett.* B **477** 285

[12] Dvali G, Gabadadze G and Porrati M, *4D Gravity on a Brane in 5D Minkowski Space* 2000 *Phys. Lett.* B **485** 208

[13] Sahni V and Shtanov Y, *Braneworld models of dark energy* 2003 *J. Cosmol. Astroparticle Phys.* JCAP **03** (11) 014

[14] Maeda K, Mizuno S and Torii T, *Effective Gravitational Equations on Brane World with Induced Gravity* 2003 *Phys. Rev.* D **68** 024033

[15] Gergely L Á and Maartens R, *Asymmetric brane-worlds with induced gravity* 2005 *Phys. Rev.* D **71** 024032

[16] Keresztes Z, Gergely L Á, Nagy B and Szabó Gy M, *The luminosity-redshift relation in brane-worlds: I. Analytical results* 2007 *astro-ph/0606698*

[17] Fairbairn M and Goobar A, *Supernova limits on brane world cosmology* 2005 *astro-ph/0511029*

[18] Alam U and Sahni V, *Confronting Braneworld Cosmology with Supernova data and Baryon Oscillations* 2006 *Phys. Rev.* D **73** 084024

[19] Alam U and Sahni V, *Supernova Constraints on Braneworld Dark Energy* 2002 *astro-ph/0209443*

[20] Maartens R and Majerotto E, *Observational constraints on self-accelerating cosmology* 2006 *Phys. Rev.* D **74** 023004

[21] Koyama K and Maartens R, *Structure formation in the DGP cosmological model* 2006 *J. Cosmol. Astroparticle Phys.* JCAP **06** (01) 016

[22] Ries A G, Strolger L-G, Tonry J et al, *Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution* 2005 *Asrophys. J.* **607** 665

[23] Astier P, Guy J, Regnault J N et al., *The Supernova Legacy Survey: Measurement of Ω_M, Ω_Λ and w from the First Year Data Set* 2006 *Astron. Astrophys.* **447** 31

[24] Lazkoz R, Maartens R and Majerotto E, *Observational constraints on phantom-like braneworld cosmologies* 2006 *Phys. Rev.* D **74** 083510

[25] Dąbrowski M P, Godłowski W and Szydłowski M, *Brane universes tested against astronomical data* 2004 *Int. J. Mod. Phys.* D **13** 1669

[26] Fay S, *Branes: cosmological surprise and observational deception* 2006 *Astron. Astrophys.* **452** 781

[27] Long J C et al., *New experimental limits on macroscopic forces below 100 microns* 2003 *Nature* **421** 922

[28] Gergely L Á and Keresztes Z, *Irradiated asymmetric Friedmann branes* 2006 *J. Cosmol. Astroparticle Phys.* JCAP **06** (01) 022

[29] Germani C and Maartens R, *Stars in the braneworld* 2001 *Phys. Rev.* D **64** 124010

[30] Maartens R, Wands D, Bassett B A and Heard I P C, *Chaotic Inflation on the brane* 2000 *Phys. Rev.* D **62** 041301(R)

[31] Ichiki K, Yahiro M, Kajino T, Orito M and Mathews G J, *Observational Constraints on Dark Radiation in Brane Cosmology* 2002 *Phys. Rev.* **D 66** 043521

[32] Bratt J D, Gault A C, Scherrer R J and Walker T P, *Big Bang Nucleosynthesis Constraints on*

Brane Cosmologies 2002 *Phys. Lett.* B **546** 19

[33] Springel V, White S D M, Jenkins A et al, *Simulating the joint evolution of quasars, galaxies and their large-scale distribution* 2005 Nature **435** 629

[34] A. Chamblin, A. Karch and A. Nayeri, *Thermal Equilibration of Brane-Worlds* 2001 *Phys. Lett* B **509** 163

[35] Langlois D, Sorbo L and Rodríguez-Martínez M, *Cosmology of a brane radiating gravitons into the extra dimension* 2002 *Phys. Rev. Lett.* **89** 171301

[36] Pal S, *Structure formation on the brane: A mimicry* 2006 *Phys. Rev.* D **74** 024005

[37] Hebecker A and March-Russell J, *Randall–Sundrum II cosmology, AdS/CFT, and the bulk black hole* 2001 *Nuclear Physics* B **608** 375

Leeper E, Maartens R and Sopuerta C, *Dynamics of radiating braneworlds* 2004 *Class. Quant. Grav.* **21** 1125

Gergely L Á, Leeper E and Maartens R, *Asymmetric radiating brane-world* 2004 *Phys. Rev.* D **70** 104025

Jennings D and Vernon I R, *Graviton emmission into non-Z2 symmetric brane world spacetimes* 2005 *J. Cosmol. Astroparticle Phys.* JCAP **05** (07) 011

Langlois D, *Is our Universe brany?* 2006 *Progress of Theoretical Physics Supplement* **163** 258

[38] Riess A G, Filippenko A V, Challis P et al., *Observational evidence from Supernovae for an Accelerating Universe and a Cosmological Constant* 1998 *Astrophys. J.* **116** 1009

[39] Schmidt B, 2005 in: *The new cosmology,* Proceedings of the 16th Int. Summer School 2003 Singapore World Scientific

[40] Tonry J L, Schmidt B P, Barris B et al., *Cosmological Results from High-z Supernovae* 2003 *Astrophys. J.* **594** 1

[41] Riess A G, Strolger L-G, Casertano S et al., *New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z > 1$: Narrowing Constraints on the Early Behavior of Dark Energy* 2007 to appear in Astrophys. J. **656** *astro-ph/0611572*

[42] Garcia-Bellido J, *Cosmology and Astrophysics* 2005 *astro-ph/0502139*

[43] Kolb E W and Turner M S, *The Early Universe* 1990 Adisson Wesley

[44] Tao C, *Evidence for physics beyond LCDM?* Talk given at the Cosmology Workshop: Cosmology and Astroparticles at Université Montpellier 2 France November 23^{rd}-24^{th} 2006

[45] Nesseris S, Perivolaropoulos L, *Tension and Systematics in the Gold06 SnIa Dataset* 2006 *astro-ph/0612653*